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VIEWPOINT

The Anderson model: why does it continue to be so fascinating?

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The Anderson model [1] of an impurity state in a conductor dates from 1961. A very important development occurred in 1966 when Schrieffer and Wolff [2] showed that the Kondo Hamiltonian could be derived from the Anderson model in an appropriate limit. In this comment we address the questions:

- What are the Anderson and Kondo problems?
- What physical systems do they describe and why have they dominated many-body physics for so long?
- Why are they so hard—particularly for the so-called lattice problems?
- What is new now that makes the solution of the impurity problem so relevant?

A full answer to all these questions (except the last one) is contained in the excellent book by Hewson [3]; this is a very brief non-mathematical summary.

The Anderson model starts from a model of non-interacting Fermi gas of electrons hybridizing with the conduction electrons. If the localized energy is in a band gap then it is sharp—as for a gap state in a semiconductor. If it hybridizes with the conduction electrons to give a localized energy that is in a band, then we have a resonance. In this case any electron placed at the impurity site will stay, on average, for a time τ before it rejoins the conduction band and moves off into the rest of the sample. By the uncertainty principle this means that the state acquires an energy width Δ. There are many examples of such resonances in condensed matter physics. For example, if a heavy atom is substituted in a solid, then there will be a phonon resonance that is associated with the characteristic vibration frequency of such an atom.

In a metal, only states that lie below the Fermi level are occupied and it is convenient to measure the impurity resonance energy relative to the Fermi level. For the resonance to be essentially fully occupied, its energy ε_0 must lie below the Fermi level by an amount that exceeds its width, Δ. Indeed it is very straightforward to show [3] that the occupation of such a resonance is given by $2/\pi \cot^{-1}(\varepsilon_0/\Delta)$.

For $|\varepsilon_0| \gg \Delta$ the original localized level lies far below the Fermi level, so it has an extremely high probability of being occupied. In the opposite limit such that the localized state lies exactly at the Fermi level, the occupation of the localized state is then exactly $1/2$. All of this was known from the work of Koster and Slater [4] among others from the 1950s. This theory is linear and hence can be evaluated to any desired accuracy. It works extremely well

for neutral bosonic excitations such as phonons or quasi-localized states in photonic lattices, but fails for electron systems because of the electron–electron repulsion felt by the electrons in the impurity state as was realized by Anderson [1] who introduced a very important extra piece of physics into this model.

Anderson was interested in the question of why some impurities carried a magnetic moment in some metals but not in others. He showed that this arose because of the Coulomb repulsion between electrons on the localized state. The localized state is a unique orbit and so the Pauli exclusion principle forbids the double occupation of the localized state by two electrons of the same spin. The Coulomb repulsion introduces a term that makes double occupation by two electrons of opposite spin unfavourable. This causes the localized state to be occupied by one electron with a free spin and hence be magnetic.

We now have all the ingredients of the Anderson Hamiltonian: there is a localized energy level at energy ε_0 that hybridizes to give a state of width Δ and a repulsion, U, between two electrons of opposite spin if they occupy the localized orbital.

It costs an energy ε_0 to add an electron to an empty impurity site and an energy $\varepsilon_0 + U$ to add an electron to a site where there is already an electron. If we choose $\varepsilon_0 - U/2$, then the problem is symmetric because the two energies lie equidistant from the Fermi energy. In this case there is a particle–hole symmetry and the impurity site is always occupied by exactly one electron independent of the value of U. This is known as the symmetric Anderson model.

The Kondo model is equivalent to the Anderson model in the limit where the localized state energy, ε_0, lies far below the Fermi level but the energy $\varepsilon_0 + U$ is far above the Fermi level. In this case the localized site is overwhelmingly likely to be occupied by one electron. This one electron carries a spin which is free to change its orientation and hence this gives resonance scattering right at the Fermi level. An inelastic scattering is defined as one in which the state of the scattering particle is changed. In a Kondo system, a scattering event in which the incoming conduction electron and the localized spin both change their spin orientation is inelastic. It is well known that the electron occupation is discontinuous at the Fermi energy of a metal but rigorously this only applies at zero temperature. An inelastic scattering occurs at the Fermi level that is becoming a singular point at absolute zero. This gives a divergent perturbation theory below a temperature that is defined as the Kondo temperature and a strongly enhanced density of states just at the Fermi level. This narrow peak in the density of states at the Fermi level which emerges at low temperatures is known as the 'Kondo resonance'.

At first this was a surprise. After all, the Anderson model has two states at energies $-|\varepsilon_0|$ and $U + \varepsilon_0$ which lie far below and far above the Fermi level respectively. The Kondo Hamiltonian has singular scattering just at the Fermi level. The effect occurs because of the electron spin—a localised state with one electron will be magnetic. In the absence of any interactions the magnetic state is two fold degenerate. The weak interactions of the localized spin with the conduction electrons lead to the true ground state being a nonmagnetic singlet.

The Kondo temperature is the characteristic temperature of the problem. For $T > T_K$ we have a scattering problem and for $T < T_K$ a bound state forms in which the localized spin is exactly screened out by the conduction electrons. One of the features that makes the Anderson problem so hard is the existence of two important but very different energy scales, one characterizing the formation of the local state, ε_0, and the other the energy of the Kondo bound state, T_K. The two conduction electron bands with opposite spin are now inequivalent, as each interacts with a localized state that looks different for different spin. But the spin is not fixed and is fluctuating precisely because it is coupled to the conduction band.

The Kondo problem can be solved in the high-and low-temperature regimes and the crossover between them was found as one of the first applications of the renormalization group and later using the Bethe ansatz [3] for a rather special form of the density of states.

As in other problems of non-linear physics, the important ingredient of the problem is to recognize the correct symmetry of the solution and build it into any trial solution. Any trial solution that has the wrong symmetry is bound to fail or require a sophisticated and lengthy analysis. A powerful feature in the recent series of papers by Logan [5, 6] and his co-workers is that they have built in the local moment from the beginning.

The models discussed here describe a very rich variety of physics. This is because there are several important energies: the thermal energy (k_BT), the band structure of the conduction electrons, the position of the localized energy relative to the Fermi energy (ε_0), the Kondo temperature (T_K) and the size of the repulsion (U). The Zeeman energy should be added to this when a magnetic field is applied. The flexibility of the model means that it can be applied to physical systems as diverse as a Mn impurity in Cu and the localized states on a quantum dot interacting with a two-dimensional electron gas—anything in fact where we have a localized object where the electrons are strongly interacting with each other coupled to a wider conduction band.

The periodic Anderson model (and its near relative the Kondo lattice) describe a situation in which there is a localized state in every unit cell. Now the possibilities include almost all the problems involving strongly interacting electrons. Most notable among these are the anomalous metals containing rare earths or actinides. These materials may be thought of as having a Kondo resonance at each site, so the density of states is strongly enhanced (by factors up to 10^3) just in a narrow region exactly at the Fermi energy. In an independent electron picture the density of states is proportional to the electron effective mass and so these materials are called heavy-fermion compounds. Similar physics is believed to hold in the cuprate superconductors, the manganites that show colossal magnetoresistance and C_{60} molecules in doped fullerene.

Again it is relatively straightforward to solve these problems in the high-temperature regime using perturbation theory and sometimes in the low-temperature regime, but the crossover between the two is impossible to describe exactly. In a macroscopic system it is possible to have a real broken symmetry and exactly what symmetry is broken in the low-temperature phase of a Kondo lattice has been a source of controversy.

A new very powerful method, the dynamic mean-field theory (DMFT), has been developed very recently [7]; it gives accurate solutions of the periodic problem using the results of the impurity problem. In this method, full use is made of the density of states and the low-energy dynamics of the impurity model which contains all the important physics describing the Kondo resonance. The approximation that is made is that the localized solutions are combined together in a way that ignores some of the subtleties of the real crystal lattice. This is a good approximation because the physics is dominated by what happens as a function of energy. This method depends on solving the localized Anderson problem self-consistently with a conduction band that itself depends on the solution of the Anderson problem. Hence there is a need for a reliable and easy solution to the impurity Anderson problem.

In a recent series of articles [5] Logan and his collaborators have derived a new and much simpler solution of the Anderson problem. Their earlier papers were restricted to the symmetric Anderson model but their most recent paper [6] extends the analysis to the general case which gives it much wider applicability. Their method is physically appealing, as they build in a local moment at the beginning and then solve self-consistently. They have shown that their results compare well with the best numerical results. Since the results are analytic, they will be much easier to incorporate into the DMFT and so should lead to a very interesting and new way to investigate the heavy-fermion materials and other important problems in many-body physics.

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